

Self-gravitating non-abelian kinks as brane worlds

Alejandra Melfo,^{1,2} Roger Naranjo,¹ Nelson Pantoja,¹ Aureliano Skirzewski,¹ and Juan Carlos Vásquez¹

¹*Centro de Física Fundamental, Universidad de Los Andes, Mérida, Venezuela*

²*International Centre for Theoretical Physics, 34100 Trieste, Italy*

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We address the properties of self-gravitating domain walls arising from the breaking of an $SU(N) \times Z_2$ -symmetric theory. In the particular case of $N = 5$, we find that the two classes of stable non-abelian kinks possible in flat space have an analogue in the gravitational case, and construct the analytical solutions. Localization of fermion fields in different representations of the gauge group in these branes is investigated. It is also shown that non-abelian gauge fields localization cannot be achieved through interactions with the brane, but that in one of the two classes of kinks this localization can be implemented *via* the Dvali-Shifman mechanism.

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I. INTRODUCTION

Domain wall solutions of the Einstein-scalar field equations as a regularization of the infinitely thin Randall-Sundrum brane [1] have been extensively considered in the literature. While preserving the crucial property of localizing gravity [2–5], these thick walls have also some additional potentially useful properties, arising from the interaction of the scalar field with other fields in the theory [6–11], or simply from a non trivial internal structure [12–15]. The best known example is the addition of a Yukawa coupling to the wall’s field [6] that enables fermions of one chirality to localize on the brane, compensating for the gravitational effect that repels them. Gauge field localization is more difficult [16–20], as the scalar field is taken invariant under any gauge symmetry, in order to ensure the stability of the wall through topological arguments.

However, as pointed out in a series of papers [21–24], it is perfectly possible to build a stable domain wall in flat space out of a scalar field transforming non trivially under a non abelian group. In particular, it is shown in [23], that a quartic field theory with a $SU(N) \times Z_2$ symmetry broken down by the vev of an scalar transforming under the adjoint of $SU(N)$ has perturbatively stable domain wall solutions for odd $N > 3$.

It is only natural to ask if similar solutions can be found for the gravitating case. Such a thick brane would have interesting properties, as the scalar field will interact directly with the gauge sector, opening up the possibility of localizing the gauge fields on the brane. Inside the wall, a subgroup of the original symmetry would be preserved, allowing for localization of multiplets of fermions that could be of phenomenological interest.

In order to investigate this possibility, in section II we have taken a toy example with the minimal choice $N = 5$, and solved the Einstein-scalar field equations for a scalar field transforming in the adjoint, with a sixth-order potential. Two symmetry breaking patterns are considered, $SU(5) \times Z_2 \rightarrow SU(3) \times SU(2) \times U(1)/(Z_3 \times Z_2)$ and $SU(5) \times Z_2 \rightarrow SU(4) \times U(1)/Z_4$, finding in both cases

analytical domain wall solutions. We then study fermion localization on these solutions in section III, by considering its Yukawa interactions with fermion fields in the fundamental representation of $SU(5)$, and showing how the zero modes of some of the fermion components are localized.

Next, we turned to localization of gauge fields in section IV. First, we address the issue of whether the gauge interactions of the scalar field suffice to do that. We conclude that although a subset of the gauge fields have a localized massless mode on the brane, they necessarily interact with gauge fields that propagate in the bulk unveiling the presence of the extra dimension. This is true for any non-abelian wall arising from the breakdown of $SU(N) \times Z_2$ by a field in the adjoint representation. We then discuss the possibility of gauge field localization *via* the Dvali-Shifman (DS) mechanism [25] as a result of the so-called “clash of symmetries” (see e.g. [26] and references therein). We find that for the brane with symmetry breaking $SU(5) \times Z_2 \rightarrow SU(4) \times U(1)/Z_4$ this mechanism leads to such localization.

II. SELFGRAVITATING THICK BRANES IN $SU(5) \times Z_2$

Let us consider the $(4 + 1)$ -dimensional theory given by

$$S = \int d^4x dy \sqrt{-g} \left[\frac{R}{2} - \text{Tr}(\partial_m \Phi \partial^m \Phi) - V(\Phi) \right] \quad (1)$$

where R is the scalar curvature, g is the determinant of the metric, Φ is a field that transform in the adjoint representation of $SU(5)$ and $V(\Phi)$ is a potential invariant under the action of this group.¹

Now, since the elements of the center of $SU(N)$ transform Φ by multiplicative factors of $\exp(2\pi i/N)$, for even

¹ We use units where $\hbar = G = c = 1$

N the transformation $\Phi \rightarrow -\Phi$ is included in the center, while for odd values of N it is not [23]. It follows that for $N = 5$, in order to generate topological kink solutions that break the discrete Z_2 symmetry, the symmetry of (1) must be chosen as $SU(5) \times Z_2$.

It has been shown in [23] that in flat space and for a $SU(5)$ -invariant fourth-order potential, the vev of Φ will form perturbatively stable domain walls when the symmetry breaking pattern is

$$SU(5) \times Z_2 \longrightarrow \frac{SU(3) \times SU(2) \times U(1)}{Z_3 \times Z_2} \quad (2)$$

provided that the vevs of Φ at spatial infinity are related both by a Z_2 transformation and an $SU(5)$ one, i.e

$$\Phi(y = -\infty) = -U\Phi(y = +\infty)U^{-1} \quad (3)$$

where U is an element of $SU(5)$ and Φ depends only on the coordinate y .

On the other hand, for a fourth-order potential with a Z_2 symmetry, it is well known that there is only one other possible minimum, giving rise to the symmetry-breaking pattern

$$SU(5) \times Z_2 \longrightarrow \frac{SU(4) \times U(1)}{Z_4} \quad (4)$$

Along the lines of [23], one can find a fourth-order potential for Φ such that imposing boundary conditions as in (3) domain walls can be formed in this case also. It is straightforward to prove that these walls are stable under small perturbations.

Each of these two patterns of symmetry breaking provides only one class of stable kinks, other solutions being either unstable or equivalent.

Having thus identified the only two possibilities of constructing a stable kink from an $SU(5) \times Z_2$ theory, we turn now to the issue of incorporating gravity. In order to obtain analytical kink solutions, the potential must be chosen as a sixth-order one. Kink solutions for a sixth-order potential for a single scalar field coupled to gravity have been found in [2]. For the $SU(5) \times Z_2$ case we begin by writing the most general potential

$$V(\Phi) = -m^2 \text{Tr}[\Phi^2] + h(\text{Tr}[\Phi^2])^2 + \lambda \text{Tr}[\Phi^4] + \alpha(\text{Tr}[\Phi^2])^3 + \beta(\text{Tr}[\Phi^3])^2 + \gamma(\text{Tr}[\Phi^4])(\text{Tr}[\Phi^2]) + V_0 \quad (5)$$

where V_0 is a constant to be fixed.

Next, we follow a similar strategy as in [23], factorizing

$$\Phi = \phi_M(y)\mathbf{M} + \phi_P(y)\mathbf{P} \quad (6)$$

where \mathbf{M} and \mathbf{P} are two orthogonal diagonal generators of $SU(5)$. We demand that they satisfy the conditions

$$\begin{aligned} \text{Tr}(\mathbf{M}^2) &= \text{Tr}(\mathbf{P}^2) = 1/2 \\ \text{Tr}(\mathbf{M}\mathbf{P}) &= \text{Tr}(\mathbf{M}^3\mathbf{P}) = \text{Tr}(\mathbf{P}^3\mathbf{M}) = 0 \\ \text{Tr}(\mathbf{P}^3)\text{Tr}(\mathbf{M}\mathbf{P}^2) &= \text{Tr}(\mathbf{M}^3)\text{Tr}(\mathbf{M}^2\mathbf{P}) = 0 \\ \text{Tr}(\mathbf{M}\mathbf{P}^2)\text{Tr}(\mathbf{P}\mathbf{M}^2) &= \text{Tr}(\mathbf{P}^3)\text{Tr}(\mathbf{M}^3) = 0 \end{aligned} \quad (7)$$

Analytic solutions to the Einstein-scalar field system can be found, as in the case without gravity, if the cross-terms in (5) containing both ϕ_M and ϕ_P vanish. This requires the couplings to be related by

$$\begin{aligned} h &= -12\text{Tr}(\mathbf{P}^2\mathbf{M}^2)\lambda \\ \alpha &= \frac{4}{3} \left[\frac{\Delta_2 \text{Tr}(\mathbf{P}^4) - \Delta_1 \text{Tr}(\mathbf{M}^4)}{\Delta_1 - \Delta_2} - 6\text{Tr}(\mathbf{P}^2\mathbf{M}^2) \right] \gamma \\ \beta &= \frac{1}{6} \left[\frac{\text{Tr}(\mathbf{M}^4) - \text{Tr}(\mathbf{P}^4)}{\Delta_1 - \Delta_2} \right] \gamma \end{aligned} \quad (8)$$

where

$$\begin{aligned} \Delta_1 &\equiv 3[\text{Tr}(\mathbf{P}^2\mathbf{M})]^2 + 2\text{Tr}(\mathbf{P}^3)\text{Tr}(\mathbf{M}^2\mathbf{P}) \\ \Delta_2 &\equiv 3[\text{Tr}(\mathbf{P}\mathbf{M}^2)]^2 + 2\text{Tr}(\mathbf{M}^3)\text{Tr}(\mathbf{P}^2\mathbf{M}) \end{aligned} \quad (9)$$

and Δ_1, Δ_2 are required not to be zero simultaneously. With Φ of the form given in (6) and the space-time metric ansatz

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2; \quad \mu, \nu = 0, 1, 2, 3; \quad (10)$$

where $\eta_{\mu\nu}$ is the four-dimensional Minkowski metric, the Einstein-scalar field equations for this system are

$$\begin{aligned} 3A'' &= -(\phi_M'^2 + \phi_P'^2) \\ \frac{3}{2}A'' + 6A'^2 &= -V(\Phi_k) \\ \phi_M'' + 4A'\phi_M' &= -\frac{\partial V}{\partial \phi_M} \\ \phi_P'' + 4A'\phi_P' &= -\frac{\partial V}{\partial \phi_P} \end{aligned} \quad (11)$$

where the primes indicate differentiation with respect to the additional coordinate.

Now we impose trivial and non-trivial boundary conditions for ϕ_P and ϕ_M respectively

$$\phi_P(+\infty) = \phi_P(-\infty) \quad (12)$$

$$\phi_M(+\infty) = -\phi_M(-\infty) \quad (13)$$

It is now possible to find solutions to (11), provided there is still one more relation between the parameters in the potential and the constant V_0 is appropriately chosen. Let us write the potential as

$$\begin{aligned} V &= -\frac{m^2}{2}\phi_M^2 + \frac{\lambda_M}{4}\phi_M^4 + \frac{\gamma_M}{6}\phi_M^6 \\ &\quad -\frac{m^2}{2}\phi_P^2 + \frac{\lambda_P}{4}\phi_P^4 + \frac{\gamma_P}{6}\phi_P^6 + V_0 \end{aligned} \quad (14)$$

where the couplings $\lambda_M, \lambda_P, \gamma_M$ and γ_P are calculated after imposing the decoupling conditions (8). Then solutions are found to be

$$\Phi(y) = v [\tanh(by)\mathbf{M} + \kappa\mathbf{P}] \quad (15)$$

with a warp factor

$$A(y) = -\frac{v^2}{9} [2 \ln(\cosh(by)) + \frac{1}{2} \tanh^2(by)] \quad (16)$$

where

$$v^2 = \frac{1}{4} \left(\frac{\lambda_M}{\gamma_M} - \frac{9}{2} \right); \quad b^2 = \frac{9}{4} v^2 \gamma_M \quad (17)$$

provided that

$$m^2 = 2b^2 \left(1 + \frac{2}{3} v^2 \right) \quad (18)$$

$$V_0 = -v^4 b^2 \frac{8}{27} + \kappa^4 v^4 \left(\frac{\lambda_P}{4} + \frac{\gamma_P}{3} \kappa^2 v^2 \right) \quad (19)$$

Here a crucial role is played by the constant κ , given by

$$\kappa^2 = \frac{\lambda_P}{\lambda_M} \frac{\gamma_M}{\gamma_P} \frac{1 + \sqrt{1 + 4m^2 \gamma_P / \lambda_P^2}}{1 + \sqrt{1 + 4m^2 \gamma_M / \lambda_M^2}} \quad (20)$$

In order for the solutions to exist, κ must be a numerical factor, and this depends on the choice of \mathbf{M} and \mathbf{P} .

The potential $V(\Phi)$ takes the asymptotic values

$$V(\Phi)_{y=-\infty} = V(\Phi)_{y=+\infty} = -\frac{8}{27} b^2 v^4 \quad (21)$$

so that the space-time is asymptotically AdS_5 .

A. A thick brane with symmetry breaking

$SU(5) \times Z_2 \longrightarrow SU(3) \times SU(2) \times U(1) / (Z_3 \times Z_2)$

In order to satisfy the boundary conditions (3) and ensure stability, the first class of kinks has Φ taking the asymptotic values

$$\begin{aligned} \Phi_{\mathbf{A}}(+\infty) &\sim v \text{diag}(3, 3, -2, -2, -2) \\ \Phi_{\mathbf{A}}(-\infty) &\sim v \text{diag}(2, 2, -3, -3, 2) \end{aligned} \quad (22)$$

We take \mathbf{M} and \mathbf{P} as

$$\begin{aligned} \mathbf{M}_{\mathbf{A}} &= \frac{1}{\sqrt{40}} \text{diag}(1, 1, 1, 1, -4) \\ \mathbf{P}_{\mathbf{A}} &= \frac{1}{2\sqrt{2}} \text{diag}(1, 1, -1, -1, 0) \end{aligned} \quad (23)$$

Then solutions of the form (15-19) exist and have

$$\kappa_{\mathbf{A}} = \sqrt{5} \quad (24)$$

so that they satisfy the boundary conditions (22). This solution is the gravitational analogue of the wall found in [23].

Let us briefly review the symmetry breaking induced by this wall. At spatial infinity along the bulk coordinate, the remaining gauge symmetries H are the same, but correspond to different embeddings in $SU(5)$ as $y \rightarrow -\infty$ or $y \rightarrow \infty$. We have at $y \rightarrow \pm\infty$

$$H_{\pm}^{\mathbf{A}} = \frac{SU(3)_{\pm} \times SU(2)_{\pm} \times U(1)_{\pm}}{Z_2 \times Z_3} \quad (25)$$

Inside the wall where the vev in the \mathbf{M} direction vanishes, the unbroken group H_0 is

$$H_0^{\mathbf{A}} = \frac{SU(2)_+ \times SU(2)_- \times U(1)_M \times U(1)_P}{Z_2 \times Z_2} \quad (26)$$

where we have named the $U(1)$ factors according to their generators. The embedding is such that $SU(2)_{\pm} \subset SU(3)_{\mp}$.

B. A thick brane with symmetry breaking

$SU(5) \times Z_2 \longrightarrow SU(4) \times U(1) / Z_4$

A brane with Φ taking the asymptotic values

$$\begin{aligned} \Phi_{\mathbf{B}}(+\infty) &\sim v \text{diag}(1, 1, 1, 1, -4) \\ \Phi_{\mathbf{B}}(-\infty) &\sim v \text{diag}(-1, -1, -1, 4, -1) \end{aligned} \quad (27)$$

can be found by choosing

$$\begin{aligned} \mathbf{M}_{\mathbf{B}} &= \frac{1}{\sqrt{60}} \text{diag}(-2, -2, -2, 3, 3) \\ \mathbf{P}_{\mathbf{B}} &= \frac{1}{2} \text{diag}(0, 0, 0, 1, -1) \end{aligned} \quad (28)$$

where now

$$\kappa_{\mathbf{B}} = \sqrt{\frac{5}{3}} \quad (29)$$

An analogue of this wall for the flat space case (with a fourth-order potential) exists, although it has not been reported to our knowledge. It is not difficult however to show that it is perturbatively stable in the flat space case. It is also straightforward to show that walls built by choosing boundary conditions with different embeddings of the $SU(4) \times U(1)$ group (i.e., interchanging the position of the elements on the diagonal of, say, $\Phi_{\mathbf{B}}(-\infty)$) are equivalent to this one, corresponding to the same class of kinks.

In this case, we have at spatial infinity

$$H_{\pm}^{\mathbf{B}} = \frac{SU(4)_{\pm} \times U(1)_{\pm}}{Z_4} \quad (30)$$

and inside the wall

$$H_0^{\mathbf{B}} = \frac{SU(3) \times U(1)_M \times U(1)_P}{Z_3} \quad (31)$$

The $SU(3)$ group in $H_0^{\mathbf{B}}$ is embedded in distinct manners in the $SU(4)$ groups of $H_{+}^{\mathbf{B}}$ and $H_{-}^{\mathbf{B}}$.

III. FERMION LOCALIZATION

As is well known [6], fermions in a warped geometry are not localized on the brane, unless a Yukawa coupling with the scalar field that generates the brane forces them. In order to see what happens in the non-abelian case, we

take as an example fermions in the fundamental representation of $SU(5)$.

Consider the 5-dimensional action in a warped space-time with metric (10), for a massless spinor field Ψ in the fundamental representation of $SU(5)$ coupled to a scalar field Φ in the adjoint

$$S_\Psi = \int d^4x dy \sqrt{-g} \bar{\Psi}^I [i\Gamma^m \delta^{IJ} \nabla_m + \eta \Phi^{IJ}] \Psi^J \quad (32)$$

where I, J are $SU(5)$ indices, m, n, \dots are five-dimensional Lorentz indices, g is the determinant of the metric, ∇_m is the covariant derivative and the Γ^m satisfy the Clifford algebra. Dirac's equation reads

$$[(i\Gamma^\mu e^{-A} \partial_\mu + i\Gamma^y (2A + \partial_y)) \delta^{IJ} - \eta \Phi^{IJ}] \Psi^J = 0 \quad (33)$$

We now factorize the fermion field as usual

$$\Psi^I(x, y) = [\psi(x) f(y)]^I \quad (34)$$

and require that the four dimensional part ψ^I satisfies the Dirac equation for a massless spinor, $\gamma^\mu \partial_\mu \psi^I = 0$. For Φ given by (15), the solution for the chiral modes is straightforward

$$f_{\mathcal{L}, \mathcal{R}}^I \propto \exp[-2A(y) \mp \eta v (Q_M^I \ln(\cosh by)/b + \kappa Q_P^I y)] \quad (35)$$

where Q_M^I, Q_P^I are the charges of the I component of the fermion multiplet under M and P respectively.

Notice that the term proportional to κ can spoil localization as it diverges in either positive or negative infinity. An asymptotic analysis reveals that localization takes place only for components satisfying

$$|Q_M^I| > \kappa |Q_P^I| \quad (36)$$

Once this equation is satisfied, the sign of $Q_M(\Psi^a)$ will determine which chiral mode is localized for a given choice of the sign of the Yukawa coupling, provided it is large enough.

Take as an example solution **A**. Under the brane's symmetry group $H_0^{\mathbf{A}}$, the fermion field decomposes into a $SU(2)_+$ doublet f^+ , a $SU(2)_-$ doublet f^- , and a singlet f^0 under both. Only (one chiral mode of) the singlet component f^0 is localized on the brane, provided that

$$|\eta| > \frac{\sqrt{10}}{9} v b \quad (37)$$

As another example, consider a fermion field transforming under the the **10**-dimensional representation of $SU(5)$: it will have one chiral mode of four of its components confined in case **A**, corresponding to the bidoublet under the two $SU(2)$ factors, which is not charged under \mathbf{P}_A . In case **B**, the $SU(3)$ triplet in the fundamental gets confined.

IV. GAUGE FIELD LOCALIZATION

Now, let us consider the gauge field sector in the background of a selfgravitating non-Abelian domain wall generated by Φ

$$S_A = \int d^4x dy \sqrt{-g} \left(-\frac{1}{4} \text{Tr}(F_{mn})^2 - \mathbf{g}^2 \text{Tr}([A_m, \Phi])^2 \right) \quad (38)$$

where $F_{mn} = \partial_m A_n - \partial_n A_m + [A_m, A_n]$, A_m are the gauge fields and \mathbf{g} is the gauge coupling constant. We want to assess whether the interactions with the scalar field Φ can help to localize on the brane the massless zero modes of 5-dimensional gauge fields A_m . Since we are interested in the gauge field propagation in the additional dimension, we need to consider only the quadratic terms in the action. The effect of the terms of third and fourth order in the field A_m will be discussed later.

The linear part of the equation of motion for the gauge fields A_m is

$$\frac{1}{\sqrt{-g}} \partial_m (\sqrt{-g} g^{mp} g^{nq} (\partial_q A_p^a - \partial_p A_q^a)) - (M^2)^{ab} g^{np} A_p^b = 0 \quad (39)$$

where the mass matrix M^2 is given by

$$(M^2)^{ab} = -\text{Tr}([T^a, \Phi][T^b, \Phi]) \quad (40)$$

Notice that the symmetry breaking induced by Φ resembles the proposal of Ref.[16] (see also [19]) to localize zero mode gauge fields.

From (39), in the axial gauge $A_y = 0$, we find

$$-e^{-2A} \square A_\mu^a + e^{-2A} \eta^{\alpha\beta} \partial_\beta \partial_\mu A_\alpha^a - 2A' \partial_y A_\mu^a - \partial_y \partial_y A_\mu^a - (M^2)^{ab} A_\mu^b = 0 \quad (41)$$

and

$$\eta^{\mu\nu} \partial_\nu \partial_y A_\mu^a = 0 \quad (42)$$

We now factorize $A_\mu^a = (A_\mu(x) f(y))^a$, and require that the four-dimensional part A_μ^a satisfies a massless Proca equation, i.e. $\square A_\mu^a = 0$ and $\eta^{\mu\nu} \partial_\nu A_\mu^a = 0$ [16]. Hence

$$[(-\partial_y \partial_y - 2A' \partial_y) \delta^{ab} - (M^2)^{ab}] f(y)^b = 0 \quad (43)$$

From (40), it follows that the mass terms depends on the direction in $SU(5)$ space.

Let $(M^2)^{(n)}$ be the restriction of M^2 along a subset of generators. Along the n_{br} generators $\{T_{\text{br}}\}$ that satisfy

$$[T_{\text{br}}, \mathbf{M}] = 0, \quad [T_{\text{br}}, \mathbf{P}] \neq 0 \quad (44)$$

the mass term is constant

$$(M^2)^{(\text{br})} = c_1 \mathbf{g}^2 v^2 \mathbf{1}_{n_{\text{br}} \times n_{\text{br}}} \quad (45)$$

These generators are broken everywhere, and we have $n_{\text{br}} = 8$ for solution **A**, $n_{\text{br}} = 2$ for solution **B**.

Along the n_K generators $\{K_\pm\}$ defined as those satisfying

$$[K_\pm, \mathbf{M}] = \mp \kappa [K_\pm, \mathbf{P}] \neq 0 \quad (46)$$

it is given by

$$(M^2)^{(\pm)} = c_2 g^2 v^2 (\tanh(by) \pm 1)^2 \mathbf{1}_{n_K \times n_K} \quad (47)$$

with $n_K = 8$ for solution **A** and $n_K = 12$ for solution **B**. Here c_1, c_2 are numerical factors. For all other generators in the solutions found, M^2 is zero.

Localization of gauge fields on this brane can be readily established employing the asymptotic values for the warp factor A and the mass matrix M^2 as $y \rightarrow \pm\infty$. For a constant mass, as given by (45), we find that a normalizable solution exists which asymptotically behaves as

$$f(y)^{\text{br}} \sim \frac{1}{\sqrt{\alpha}} e^{-\alpha|y|}, \quad y \rightarrow \pm\infty \quad (48)$$

where

$$\alpha = \frac{2}{9} v^2 b \left(1 + \sqrt{1 - \frac{81 c_1}{4 b} g^2} \right) \quad (49)$$

On the other hand, for mass terms as given by (47) there are no normalizable solutions.

Thus, only vector fields with a constant mass in five dimensions, i.e. vector fields along the broken generators T_{br} , can have a normalizable zero mode on the brane. But the set $\{T_{\text{br}}\}$ is not an algebra. Hence, we have that all these normalizable zero modes, although localized on the brane, interact with fields propagating in the bulk *via* third and fourth order terms in (38).

There is however another possibility. The solutions **A** and **B** give non-trivial examples of the so-called “clash of symmetries” idea [26] to implement the DS mechanism [25] for gauge field localization. There are two different embeddings of non-abelian, confining groups outside the brane, so that the abelian gauge fields inside the brane would need to form massive glueballs in order to escape into the bulk. Non-abelian fields inside the brane would in principle have to form glueballs of a different group, so they can also become localized.

However, it is the specific nature of the breaking of $SU(5)$ that determines the possibility of obtaining localization via this mechanism. In case **A**, the solution that breaks $SU(5) \times Z_2$ to $SU(3) \times SU(2) \times U(1)/(Z_3 \times Z_2)$ asymptotically, the $SU(2)_\pm$ factors of H_0 are embedded in the $SU(3)_\mp$ part of H_\mp . Hence, $SU(2)$ glueballs inside the wall can freely escape to positive or negative infinity. The $U(1)_M$ and $U(1)_P$ gauge fields also have components in the direction of the $U(1)_\pm$ generators. This renders the DS mechanism useless here.

Case **B** in turn has an $SU(3)$ group inside the wall, embedded (in a different manner) in the two $SU(4)$ groups outside it. It is an ideal framework for the DS mechanism of gauge localization. In this respect it is similar

to the example given in [26], where a non-abelian brane in flat space breaks E_6 to $SU(5) \times U(1) \times U(1)$ within the wall. However the (perturbative) stability in this E_6 brane is rather difficult to ascertain, and its extension to the gravitational case an open question. The breaking $SU(5) \times Z_2 \rightarrow SU(4) \times U(1)/Z_4$ gives a simpler example of a stable solution allowing for implementation of the DS idea, and as we have shown, its extension to the case including gravity is straightforward.

V. SUMMARY AND OUTLOOK

We have shown that it is possible to find analytical solutions to the Einstein-scalar field equations representing a domain wall in five dimensions generated by the vev of a scalar field transforming non trivially under a non-abelian symmetry. We have found such solutions in the toy model case of the scalar being in the adjoint of $SU(5)$, with a sixth-order potential that has also a Z_2 symmetry, for the symmetry breakings $SU(5) \times Z_2 \rightarrow SU(3) \times SU(2) \times U(1)/(Z_3 \times Z_2)$ and $SU(5) \times Z_2 \rightarrow SU(4) \times U(1)/Z_4$.

These walls are capable of confining the zero mode of some components of fermions transforming as $SU(5)$ multiplets, through a Yukawa term. We have determined how localization depends on the charge of these components under the generators that define the wall.

Non-abelian gauge fields do not seem to localize on this brane through interactions with the symmetry breaking non-abelian domain wall. We have shown that although a normalizable zero mode exists for some of these vector fields, these correspond to the generators of the broken symmetries in all space. Furthermore, they necessarily interact with the remaining vector fields that propagate in the bulk, unveiling the existence of the extra dimension. It remains to be seen the effect of brane localized kinetic terms for the gauge fields, as in [17] (see also [20]).

Finally, we have shown that the nature of the breaking of $SU(5)$ determines the possibility of obtaining gauge field localization via the DS mechanism. We find that in the brane with symmetry breaking $SU(5) \times Z_2 \rightarrow SU(4) \times U(1)/Z_4$ this mechanism leads to such localization, while in the one which breaks $SU(5) \times Z_2$ to $SU(3) \times SU(2) \times U(1)/(Z_3 \times Z_2)$ asymptotically, this mechanism cannot be realized.

Several important issues need further investigation. The first is naturally the stability of these solutions. Since to our knowledge, the definition of a topological charge in the non-abelian case is an unsolved question, their perturbative stability (in which the form of the potential plays a key role) should be proved. Stability under small perturbations of various $SU(5)$ kinks in flat space has been analyzed in [23, 24], we will address the much more involved case including gravity in a forthcoming publication.

One can also worry as to what extent these toy models can be useful in a more realistic case. Namely, if one

wishes to construct a non-abelian braneworld, the symmetry group of the gauge fields localized on the brane should be more akin to the group of the standard model. This issue is also under investigation.

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- [1] L. Randall, R. Sundrum, Phys. Rev. Lett. **83** (1999) 4690-4693. [hep-th/9906064].
 - [2] O. DeWolfe, D. Z. Freedman, S. S. Gubser, A. Karch, Phys. Rev. **D62** (2000) 046008. [hep-th/9909134].
 - [3] M. Gremm, Phys. Lett. **B478** (2000) 434-438. [hep-th/9912060].
 - [4] Z. Kakushadze, P. Langfelder, Mod. Phys. Lett. **A15** (2000) 2265-2280. [hep-th/0011245].
 - [5] O. Castillo-Felisola, A. Melfo, N. Pantoja, A. Ramirez, Phys. Rev. **D70** (2004) 104029. [hep-th/0404083].
 - [6] B. Bajc, G. Gabadadze, Phys. Lett. **B474** (2000) 282-291. [hep-th/9912232].
 - [7] A. Kehagias, K. Tamvakis, Phys. Lett. **B504** (2001) 38-46. [hep-th/0010112].
 - [8] C. Ringeval, P. Peter, J. -P. Uzan, Phys. Rev. **D65** (2002) 044016. [hep-th/0109194].
 - [9] R. Koley, S. Kar, Class. Quant. Grav. **22** (2005) 753-768. [hep-th/0407158].
 - [10] A. Melfo, N. Pantoja, J. D. Tempo, Phys. Rev. **D73** (2006) 044033. [hep-th/0601161].
 - [11] Y. -X. Liu, C. -EFu, L. Zhao, Y. -S. Duan, Phys. Rev. **D80** (2009) 065020. [arXiv:0907.0910 [hep-th]].
 - [12] A. Melfo, N. Pantoja, A. Skrzewski, Phys. Rev. **D67** (2003) 105003. [gr-qc/0211081].
 - [13] D. Bazeia, C. Furtado, A. R. Gomes, JCAP **0402** (2004) 002. [hep-th/0308034].
 - [14] R. Guerrero, R. O. Rodriguez, R. S. Torrealba, Phys. Rev. **D72** (2005) 124012. [hep-th/0510023].
 - [15] R. Guerrero, A. Melfo, N. Pantoja, R. O. Rodriguez, Phys. Rev. **D74** (2006) 084025. [hep-th/0605160].
 - [16] S. J. Huber and Q. Shafi, Phys. Rev. D **63** (2001) 045010 [arXiv:hep-ph/0005286].
 - [17] G. R. Dvali, G. Gabadadze and M. A. Shifman, Phys. Lett. B **497**, 271 (2001) [arXiv:hep-th/0010071].
 - [18] S. L. Dubovsky, V. A. Rubakov, Int. J. Mod. Phys. **A16** (2001) 4331-4350. [hep-th/0105243].
 - [19] B. Batell, T. Gherghetta, Phys. Rev. **D75** (2007) 025022. [hep-th/0611305].
 - [20] R. Guerrero, A. Melfo, N. Pantoja, R. O. Rodriguez, Phys. Rev. **D81** (2010) 086004. [arXiv:0912.0463 [hep-th]].
 - [21] G. R. Dvali, H. Liu, T. Vachaspati, Phys. Rev. Lett. **80** (1998) 2281-2284. [hep-ph/9710301].
 - [22] L. Pogosian, T. Vachaspati, Phys. Rev. **D62** (2000) 123506. [hep-ph/0007045].
 - [23] T. Vachaspati, Phys. Rev. **D63** (2001) 105010. [hep-th/0102047].
 - [24] L. Pogosian, T. Vachaspati, Phys. Rev. **D64** (2001) 105023. [hep-th/0105128].
 - [25] G. R. Dvali, M. A. Shifman, Phys. Lett. **B396** (1997) 64-69. [hep-th/9612128].
 - [26] A. Davidson, D. P. George, A. Kobakhidze, R. R. Volkas and K. C. Wali, Phys. Rev. D **77** (2008) 085031 [arXiv:0710.3432 [hep-ph]].